

A school bus routing and scheduling problem with time windows and possibility of outsourcing with the provided service quality

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Abstract

A school bus problem is one of the most important logistical issues faced by governments and school administrations, which is widely depends on route optimization and scheduling. This study presents a mathematical model for the single-load school bus problem for urban areas, which considers bus stop selection, bus route generation, and school bus scheduling simultaneously. This model consider the possibility of demand outsourcing (e.g., students using parallel systems for transportation) and time windows in the school bus routing and scheduling (SBRS) problem. Each vehicle transports students from a bus stop to their school under the given constraints. Our goal is to maximize the transportation firm's profits and increase the quality of service provided for students. To validate the presented model, the problem is solved by GAMS software, and the results are shown and analyzed. Finally, some sensitivity analyses are performed to show the efficiency of the presented model.

Keywords: School bus routing, School bus scheduling, Bus stop selection, Demand outsourcing, Optimization algorithm

1 Introduction

Transferring students from their homes to schools and after that from schools to their houses is one of the most important issues faced by school boards and the transportation companies in the community that must be performed in the safest and most economical way possible. Unlike the United States, Iran's government does not provide free school bus services for students of any grade. So private transportation firms offer these services for students, who are willing to pay fees for them.

A few weeks before school starts, the school administration provides a list of students' addresses interested in using the school bus service to the transportation firm.

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Based on these addresses, the firm identifies some potential bus stop locations for students.

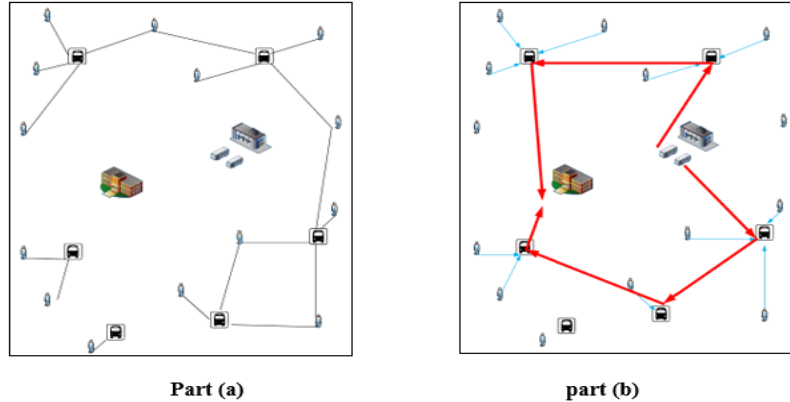


Fig. 1. An example of a school bus routing problem with potential bus stops considering the possibility of student's reluctance in selecting the transportation system.

School bus problems fall into five main sub-problems: data preparation, assigning students to bus stations (i.e., bus stop selection), bus route generation, school bus scheduling, and adjustment of school bell time. This study focuses on a family of school bus routing and scheduling problems that look simultaneously for bus stops for assigning students to them from a set of potential bus stops and designing optimal bus routes visiting such stops and scheduling those trips. This particular problem is more considered in the urban areas rather than in the rural surroundings because, in urban areas, there are designated bus stops by considering maximum walking distance for students, but in rural areas, buses pick-up students from their own homes.

Fig. 1 shows an example of a school bus routing problem. In Part (a), an input instance where students, potential stations, school, depot, and students' access to bus stations are shown. The black line shows that each student has access to which bus stations. Part (b) shows a feasible solution (not exactly optimal solution), where red arrows show the sequence of trips assigned to each school bus, and blue arrows show the station that each student must go to board on the bus. As can be seen in Part (b), some students can choose none of the activated bus stations. This can be due to a lack of economic justification or because of the limited capacity of vehicles.

School bus transportation costs generally include bus acquisition costs, driver costs, and operating costs. Bus acquisition costs and driver costs directly depend on how many buses we use for student transportation, and the operations costs depend on the total service time of buses ([1]). The income of the firm depends on the number of students it serves. The objective of our proposed problem is to design a transportation plan to increase served students and, at the same time, find the best routes to transfer them to maximize the firm's profit.

This paper addresses a school bus routing and scheduling problem that simultaneously choose a set of bus stops to visit among a set of potential bus stops, assigns students to the selected stops, designs optimal routes for each bus and schedules trips to pick up the students from the assigned stops on time. To formulate the proposed SBRS problem, a mixed-integer non-linear programming (MINLP) model is presented. This study aims to suggest an alternative model for the school bus operation system that simultaneously increases total profit for transportation firms, improves service quality for students, and addresses parents' complaints, focusing on decreasing travel time and walking distance for students to reach the bus stop. Although there are some approaches in the literature that can be used to reduce the total traveling time of school bus, but they do not specifically consider the quality of service for each student and to the best of our knowledge, it was not properly dealt with by the school bus routing literature. In addition, we also consider a combination of certain constraints on the resources used by each route.

The paper is organized as follows. Section 2 describes a literature review on the school bus problem. Section 3 presents our developed MINLP model. In Section 4, we present the computational experiments carried out to evaluate the model. Concluding remarks are drawn in the last Section 5.

2 Literature Review

Due to the importance of this issue in educational system, the problem has been receiving increasing attention in the vehicle routing literature. In this section, we review developed methods and techniques capable of being applied to this problem. We refer to [2] for detailed reviews of the school bus problem and its several variants.

The school bus routing problem (SBRP), depending on whether students from multiple schools can use the same bus at the same time or not, can be divided into single-load problem or mixed-load problem ([3],[4],[5]). In recent years there has been a great deal of researches with different methods and solutions to solve the school bus routing problem (SBRP). In this study we consider SBRP with single-load of students, because although from the economical point of view, single-load case increases the number of buses needed and as a result, total costs for transportation increases, but in practice, many schools do not allow the use of mixed-load due to social or parent concerns ([1]).

The school bus routing problem falls into a larger class of problems, called the vehicle routing problem or VRP. But it has some qualitative differences with the simple vehicle routing problem because students are not like packages that are simply picked up from a location and delivered elsewhere. [6] modeled their bus route generation problem and tried to minimize the total travel time. They propose a hybrid column generation and clustering method to solve their problem. Some studies considered bus route generation and bus stop selection at the same time being a possible way to optimize school bus problems. [7] divided the problem into two steps. In the first step, they identified the most suitable concentration points of students, and in the second step, they find the optimal routes to serve those identified stops. They tried to minimize the total walking time of students to their assigned bus stops. [8] expanded a previous study [9] by introducing some additional constraints for their proposed model like the minimum

number of students needed to pick up by each vehicle to create a route, to minimize transportation firm's total costs and total students walking distance. A branch-and-price algorithm based on a set partitioning formulation for this problem was used to deal with constraints on resource consumption. [10] proposed a bi-objective mixed-integer linear programming formulation and tried to minimize the total number of vehicles needed and the students' average riding time, considering one type of school bus. [2] developed a bi-level mathematical model for the SBRP and designed a transportation system while considering the possibility that the students can choose to use the system or not and tried to maximize the transportation firm's total profit. They considered the designer of the school bus transportation system to be the upper-level decision-maker, and the students are the lower level decision-makers. They consider that Students can decide to use the provided service for them at the determined bus stop or not. [11] considered the limitation of resources (e.g., vehicles and drivers) in their routing problem. Due to this limitation, the company may not be able to provide service to some customers. They used a hierarchical objective function to maximize the number of provided services to customers as the primary objective and minimize the transportation costs as secondary.

Another class of school bus problem is the ones that consider both school bus route generation and route scheduling (SBRS) at the same time. [12] introduced the concept of the time window in the traveling salesman problem. Results showed that it significantly reduces the state space and the number of state transitions. [13] presented a model to reduce the number of vehicles needed to transport students to schools and minimize the sum of all deadhead trips in their routing and scheduling problem. They considered that each bus must arrive at the bus stop on a certain time after serving all assigned students and reach the school within a fixed time window. [14] presented a school bus scheduling problem to optimize bus schedules to serve all of the given trips with consideration of the school time window wherein all bus trips are given. They proposed two assignment problem based on exact approaches and a heuristic algorithm for some special cases and general cases. [15] considered a heterogeneous fleet of multiple vehicle types for their vehicle routing problem with time windows. Their objective was to minimize transportation costs while providing service for a set of customers within their time windows. They proposed some exact algorithms under the cluster-first and route-second approach to solve their VRP. [16] introduced the on-time arrival probability in their routing and scheduling problem as a constraint. Namely, all school buses must get to school within a pre-determined arrival time with a required reliability level. They formulated and solved their problem by a heuristic approach, which is a combination of a proposed Route decomposition algorithm and ant colony optimization.

Although in most previous studies, bus stop selection, bus route generation, and school bus scheduling are considered independently, all problems are highly interrelated. To fill the literature gaps, this study formulates a new MINLP for the SBRS problem by considering a location-allocation-routing strategy and assigning students (not particularly all of them) to those selected bus stops simultaneously. The main new suppositions that we considered in our proposed model are students' walking distance, students' total traveling time in the vehicle, time window, and minimum student number

required to create a route. To the best of our knowledge, this is the first study that considers the quality of service provided for students with the possibility of outsourcing (students using parallel systems).

3 Problem description

The school bus routing and scheduling (SBRS) problem with a heterogeneous fleet and time windows can be modeled as follows. Let V , S , and K be the set of potential bus stops, students, and buses, respectively. The main idea of the problem is to design and schedule a network to maximize the profit of the transportation firm by considering students' choice to use the service or not. Students prefer to walk fewer distances to reach the station and board the bus as late as possible, considering that they get to school on time. The time window is the possible time range of bus arrival time to station i . a_i is the earliest acceptable time for students to board the bus at the station i and b_i is the latest time for students to board the bus so that after the service, the school bus arrives at school on time. There may be situations where assigning a service trip to a student is not profitable for the firm, or the boarding station designated by the firm for the student may not be accessible to him/her. In this case, the student can use other ways to get to school. Clearly, the firm must provide service to more students for maximizing its profits, and it is only possible by allocating more stations and routes to each vehicle.

Index sets:

V	Set of potential bus stops, $V \in \{1, \dots, v\}$
K	Set of vehicles, $K \in \{1, \dots, k\}$
S	Set of students, $S \in \{1, \dots, s\}$
$D = \{0\}$	Index for the depot
$S = \{n\}$	Index for the school

Parameters:

CB_k	Capacity of the k -th bus
d_{il}	The distance of the l -th student from the i -th potential station (in the case of lack of access, the big- M distance is considered)
O_{il}	1 if the l -th student has access to the i -th potential station; 0, otherwise
a_i	Earliest possible start time of the visit for station i
b_i	Latest possible start time of the visit for station i
t_{ij}	Traveling time between node i and node j
φ	Fines of students to be on the bus per minute
C_{ij}	Cost of the travel along arc $i - j$
FC_k	Fixed costs of acquisition of bus k in the time period
CI	The income obtained from providing service for each student
CW	Costs of students walk per unit of length
CL	finer for lack of service to each student
bt	Required time for a student to board the bus
st	School' start time

tt	Maximum allowable traveling time for students
CN_k	Minimum number of students assigned to vehicle k to use the vehicle on the network
M	A large number

Decision variables:

x_{ijk}	1 if bus k traverses the arc from i to j ; 0, otherwise
y_{ik}	1 if bus k visits stop i ; 0, otherwise
z_{ilk}	1 if student l is picked up by bus k at stop i ; 0, otherwise
at_{ik}	Service arrival time at station i by bus k
e_k	1 if bus k is needed for transportation; 0, otherwise

Our MINLP model for the SBRS problem is as follows:

$$\begin{aligned} \text{Max } Z = & CI \times \sum_{i \in V} \sum_{l \in S} \sum_{k \in K} z_{ilk} - \sum_{i \in V} \sum_{j \in V} C_{ij} \sum_{k \in K} x_{ijk} \\ & - CW \times \sum_{i \in V} \sum_{l \in S} d_{il} \sum_{k \in K} z_{ilk} \\ & - CL \times \sum_{l \in S} \left(1 - \sum_{i \in V} \sum_{k \in K} z_{ilk} \right) - \sum_{k \in K} FC_k \times e_k \\ & - \varphi \times \sum_{l \in S} \sum_{i \in V} \sum_{k \in K} z_{ilk} \times (st - at_{ik}) \end{aligned} \quad (1)$$

s.t.

$$\sum_{i \in DUUV} x_{ijk} = \sum_{i \in VUS} x_{jik} = y_{jk} \quad ; \forall i \in V \quad (2)$$

$$\sum_{k \in K} y_{ik} \leq 1 \quad ; \forall j \in V, k \in K \quad (3)$$

$$\sum_{i \in V} \sum_{l \in S} z_{ilk} \leq CB_k \quad ; \forall k \in K \quad (4)$$

$$\sum_{i \in V} \sum_{k \in K} z_{ilk} \leq 1 \quad ; \forall l \in S \quad (5)$$

$$\sum_{k \in K} z_{ilk} \leq O_{il} \quad ; \forall i \in V, l \in S \quad (6)$$

$$z_{ilk} \leq y_{ik} \quad ; \forall i \in V, l \in S, k \in K \quad (7)$$

$$at_{ik} + t_{ij} + \sum_{i \in V} \sum_{l \in S} bt \times z_{jlk} - at_{jk} \leq (1 - x_{ijk})M \quad ; \forall i, j \in V, i \neq j, k \in K \quad (8)$$

$$a_j \sum_{i \in DUUV} x_{ijk} \leq at_{jk} \leq b_j \sum_{i \in DUUV} x_{ijk} \quad ; \forall j \in V, k \in K \quad (9)$$

$$\sum_{j \in V} x_{0jk} \leq 1 \quad ; \forall k \in K \quad (10)$$

$$\sum_{i \in V} x_{ink} \leq 1 \quad ; \forall k \in K \quad (11)$$

$$\sum_{i \in V} \sum_{j \in V \cup S} x_{ijk} \times t_{ij} + \sum_{i \in V} \sum_{l \in S} bt \times z_{ilk} \leq tt \quad ; \forall k \in K \quad (12)$$

$$\sum_{j \in V} x_{0jk} = e_k \quad ; \forall k \in K \quad (13)$$

$$\sum_{i \in V} \sum_{l \in S} z_{ilk} \geq CN_k \times \sum_{j \in V} x_{0jk} \quad ; \forall k \in K \quad (14)$$

$$x_{ijk} \in \{0,1\} \quad ; \forall i, j \in V, i \neq j, k \in K \quad (15)$$

$$z_{ilk} \in \{0,1\} \quad ; \forall i \in V, l \in S, k \in K \quad (16)$$

$$y_{ik} \in \{0,1\} \quad ; \forall j \in V, k \in K \quad (17)$$

Objective function (1) maximizes the profit of the firm. The first part calculates the sum of incomes gained by providing service to students. The second part includes traveling costs between nodes. The third part includes the students walking costs from their home to the nearest activated bus stop if the bus has enough capacity to board them at that station. Fourth part includes fines for lack of providing service for students. The next part consists of the acquisition cost for each school bus for the school year, and the last term is for increasing the quality of the service provided for students, and it minimizes the time difference between school start time and boarding time of students as much as possible. Constraints (2) is the flow balance constraint, which ensures that if $y_{jk} = 1$, then vehicle k must enter and exit the station and if $y_{jk} = 0$, then vehicle k will not be allowed to visit station j . Constraint (3) ensures that each station will be visited at most once. Constraint (4) guarantees that the number of students allocated to bus k does not exceed vehicle k th capacity. Constraint (5) assigns students to a bus station and if providing service to a student is not economical for the transportation firm, $\sum_{i \in V} \sum_{k \in K} z_{ilk} = 0$. Constraint (6) shows the level of access for each student to each station. Constraint (7) ensures that if k th bus does not visit a station, no student board on that bus on that station. Constraint (8) states that when bus k serves station j followed by station i , arrival time at stop j is greater than or equal to the arrival time from stop i plus the travel time between i and j plus boarding time of students on station i . Constraints (9) are the time window constraints. Constraints (10) and (11) show that all school buses start their trips from depot and after traveling between a station visit the school. Constraint (12) is for quality of service to students, and ensures that traveling time in the bus for each student is lower than the allowable ride time for students. Constraint (13) shows that whether we use bus k or not. Constraint (14) guarantees for using school bus k , at least CN_k students must be assigned to it. Constraints (15) – (17) require that all decision variables are binary.

4 Computational Experiments:

It is obvious that the activation of more potential bus stops will increase transportation and routing costs, but on the other hand, the company can increase its revenues by providing services to more students. The optimal solution can be found by considering interactions between allocating more students to more bus stops and transportation costs and fins. A small-sized problem with 10 students, five potential bus stops and three types of vehicle is presented to validate our proposed model. Students' access to

bus stations and solution representation for this problem can be seen in Fig. 2. We use GAMS software [17-26] with the Baron solver and executed on an AMD FX-7600P CPU 2.7GHz up to the 3.6 GHZ system with 8 GB RAM. As can be seen, the best solution is obtained by providing services for 9 out of 10 students by only two vehicles in four bus stops, and one potential bus stop is not activated.

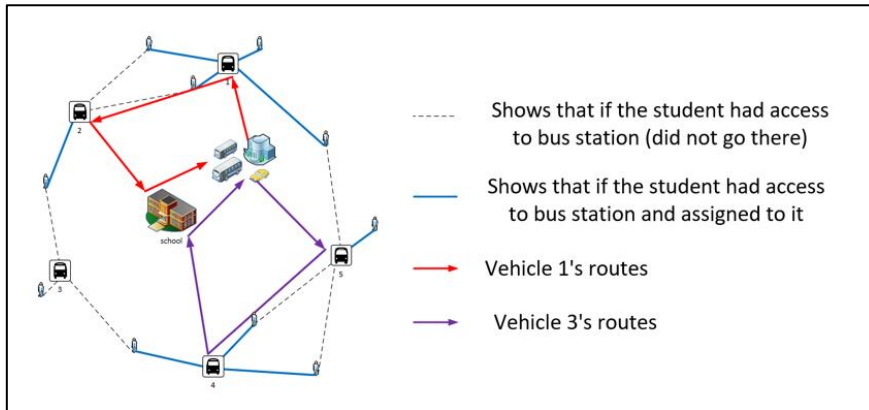


Fig. 2. Solution representation for small size problem.

In this section, we also try to determine the impact of input parameters on the system's optimized solution in a small sized problem through a sensitivity analysis. For this purpose, some of the important parameters are selected and their effects on the output of the problem is shown. In the first stage, a sensitivity analysis is done on the impact of maximum allowable time that students can be on the bus (tt) on the total gained profit of the transportation firm. The obtained results can be seen in Fig. 3. As was expected, by increasing tt , vehicles can visit more bus stops and board more students and Thus by fewer vehicles, transportation firm can provide service to more students. For example, if this parameter increased by 30%, the best answer is to visit all bus stops and board all students. Or when we decreased that parameter by 15%, the firm's total profit decreased by an amount of 44%.

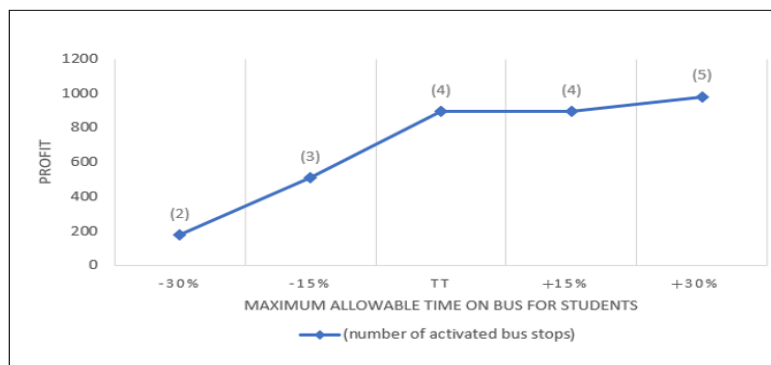


Fig. 3. Sensitivity analysis on the impact of maximum allowable time (tt) for students on the objective function.

Some sensitivity analyses are also done on CW and CL , and their effects on the number of students using provided service. As evident in Fig. 4, by increasing CW fine, number of students willing to use provided services decreases. By considering this cost to be very low, all students are willing to use the transportation firm's services. It can also be seen that CL plays an important role in solving the problem. If we consider a high value for this parameter, the transportation firm tries to provide service to most of the students.

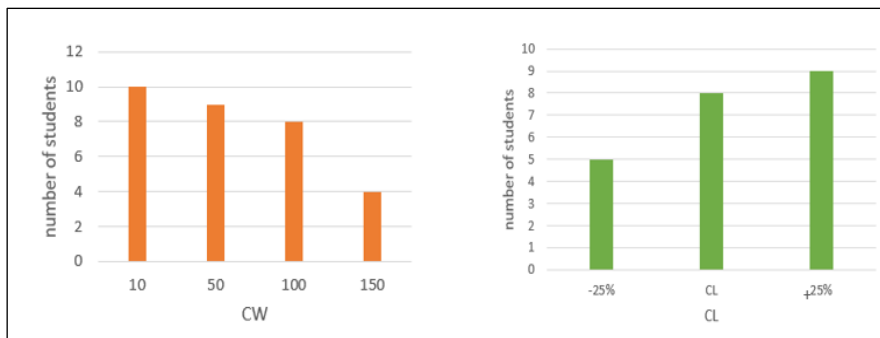


Fig. 4. Sensitivity analysis on the impact of fines for students' walk per unit of length (CW) and lack of service to each student (CL) on the total number of students that use service.

5 Conclusions and future research

To enrich a school bus routing and scheduling (SBRS) problem, we have presented a mathematical model built on the previous proposed models in the literature. This model has combined the bus stop selection and bus route generation considering maximum allowable traveling time for students, the earliest and latest pick-up time for students in bus stops, and a heterogeneous fleet of vehicles. We have validated this model by an example in GAMS software. Applying this model in a real-case situation and investigating results or using more efficient methods (e.g., meta-heuristics) and comparing efficiency of their results with some statistical analysis are recommended for future studies. It is very common to have some bus drivers work more than others because of the bus stops assigned to them. In strong unionized areas, this can lead to labor complains by drivers and be a source of nuisance to public managers. Hence, a multi-objective model that minimizes the costs and the distance unbalance among drivers and simultaneously maximizes the quality of service provided for students can also be investigated in future studies.

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